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Interaction of a slow monopole with a hydrogen atom

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Abstract

The electric dipole moment of the hydrogen-like atom induced by a monopole moving outside the electron shell is calculated. The correction to the energy of the ground state of the hydrogen atom due to this interaction is calculated.

As is well known [1] the interaction of a monopole with the atoms of matter is fundamental to the principle of operation of a number of detectors (scintillators, plastics, emulsions, etc.) used in the experimental search for magnetic charges. In this connection the interaction of a monopole with an atom has been fairly well studied [2]–[4]. Unfortunately, for the process of ionization of an atom by a monopole, there is a large natural background, which makes it difficult to identify genuine monopole tracks [5]. Therefore searches are continuing for effects that make it possible to distinguish a monopole from a heavy nucleus in interaction with atoms.

For example, in Ref.6 an estimate was obtained for the probability of excitation of an atom by a monopole on the basis of a quantum-mechanical analog of the Callan-Rubakov

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effect that occurs in the passage of a monopole through an electron. Despite its specific character, this process is obviously not dominant in the overall picture of the interaction of a monopole with an atom.

It was noted in Ref.4, that the interaction of a monopole with an atom is also characterized by a special spatial asymmetry associated with the space parity nonconservation in the theory with magnetic charge [7], [8]. These effects have been studied for the model example of a charge-dyon bound system [9]–[12].

In the present note we calculate the electric dipole moment of the hydrogen-like atom induced by a monopole moving outside the electron shell (see Fig.1) as well as the correction to the ground state energy of a hydrogen-like atom.

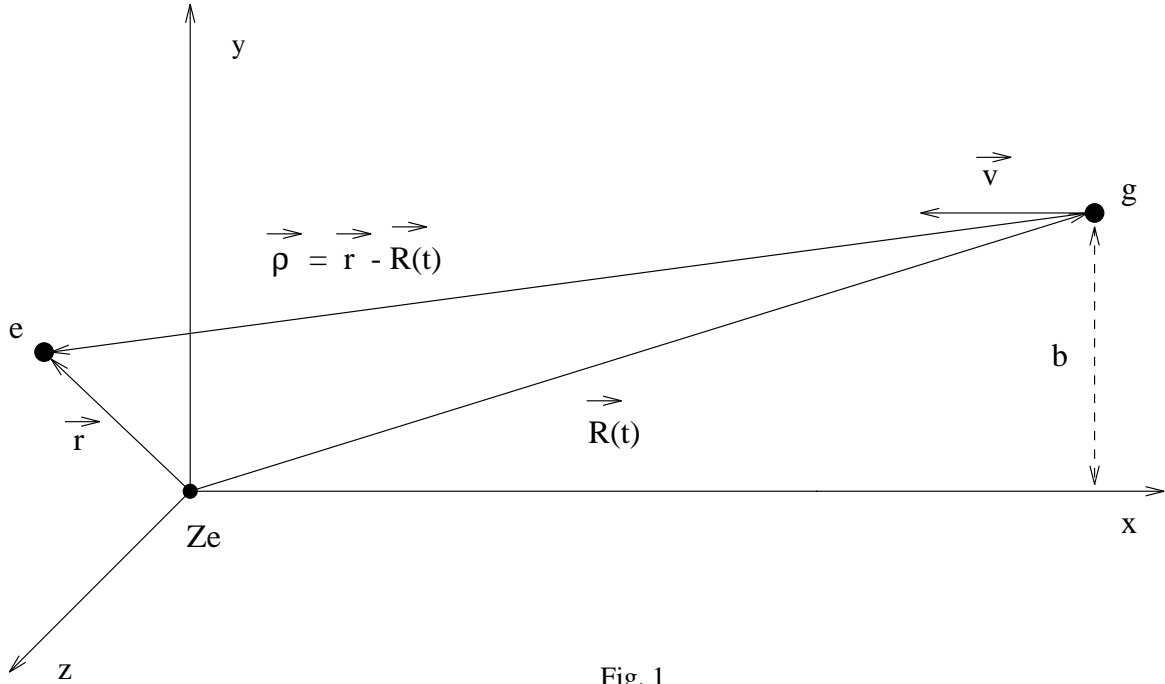


Fig. 1

Suppose that a nucleus with charge $Q = Ze$ is at rest at the origin, and that a monopole with a magnetic charge g is incident on it with impact parameter b and constant velocity $\mathbf{V} = -iv$. Let

$$\mathbf{R}(t) = ivt + \mathbf{j}b$$

be the coordinates of the monopole in the system associated with the nucleus, m the electron mass,

$$\mathbf{r} = ix + jy + kz$$

the electron coordinates, and

$$\boldsymbol{\rho} = \mathbf{r} - \mathbf{R} = (x - vt)\mathbf{i} + (y - b)\mathbf{j} + z\mathbf{k},$$

a vector directed from the monopole to the electron. Furthermore, we define

$$\varphi = \arctan \frac{\rho_y}{\rho_x} \quad \text{and} \quad \theta = \arccos \frac{\rho_z}{\rho}$$

as the azimuthal and polar angles of the electron in the coordinate system moving together with the monopole.

In a first approximation, we can assume that the monopole only excites the electron states, but does not disturb the relatively massive nucleus. In order to calculate the probability of the corresponding transitions, we note that the Hamiltonian operator describing this system (here and in what follows, $\hbar = c = 1$) is

$$\begin{aligned} H &= \frac{1}{2m}(\nabla - ie\mathbf{A}^D)^2 - \frac{eQ}{r} \\ &= \frac{1}{2m}\boldsymbol{\Pi}_r^2 + \frac{\mathbf{L}^2}{2mr^2} - \frac{eQ}{r} + W_1 + W_2, \end{aligned} \quad (1)$$

where

$$\mathbf{A}^D = g \frac{(1 - \cos \theta)}{\rho \sin \theta} \hat{\boldsymbol{\varphi}}$$

is the Dirac monopole potential, $\boldsymbol{\Pi} = \nabla - ie\mathbf{A}^D$, $\boldsymbol{\Pi}_r = 1/2(\hat{\mathbf{r}}\boldsymbol{\Pi} + \boldsymbol{\Pi}\hat{\mathbf{r}})$. Here we take into account that $(\nabla\mathbf{A}^D) = (\boldsymbol{\rho}\mathbf{A}^D) = 0$ and used the definition of the operator of total angular momentum of the electron, $\mathbf{L} = [\mathbf{r}, \boldsymbol{\Pi}]$.

The operator (1) differs from the Hamiltonian of a hydrogen-like atom by the terms

$$\begin{aligned} W_1 &= \frac{e}{m}(\mathbf{A}^D \nabla) = \frac{i\mu}{m\rho^2(1 + \cos \theta)} \frac{\partial}{\partial \varphi}, \\ W_2 &= \frac{e^2}{2m}(\mathbf{A}^D)^2 = \frac{\mu^2}{2m\rho^2} \frac{\sin^2 \theta}{(1 + \cos \theta)^2}, \end{aligned} \quad (2)$$

which describe the interaction of the atomic electron with the monopole (here $\mu = eg$).

If the monopole passes sufficiently far from the atom, so that $\varepsilon = r/R \ll 1$, the probability of a transition of the electron from the initial state $|n\rangle$ to a state $|m\rangle$ is determined by the matrix element $\langle m|W|n\rangle$, where $W = W_1 + W_2$ can be regarded as a perturbation operator.

Now we take into account that for small ε

$$\cos \theta \approx \frac{z}{R} + O(\varepsilon^2), \quad \frac{1}{\rho^2} \approx \frac{1}{R^2} + \frac{1}{R^4}(2vtx + 2by) + O(\varepsilon^3) \quad (3)$$

and hence

$$\begin{aligned}
W_1 &\approx \frac{\mu}{mR^2} \left((b-y)P_x + (x-vt)P_y \right) + O(\varepsilon^2) = \\
&= \frac{\mu}{mR^2} \left(iL_z + (bP_x - vtP_y) \right) + O(\varepsilon^2); \\
W_2 &= \frac{\mu^2}{2mR^4} \left((b-y)^2 + (x-vt)^2 \right) + O(\varepsilon^3) = \\
&= \frac{\mu^2}{2mR^2} + \frac{\mu^2}{mR^4} \left(\frac{(x^2 + y^2)}{2} - (vtx + by) \right) + O(\varepsilon^3). \tag{4}
\end{aligned}$$

It is worth noting, that the operator W Eq.(2) represents a mixture of a scalar and a pseudoscalar. It means, that among dipole transitions in the hydrogen atom spectrum, stipulated by external monopole perturbation, there are transitions which violate parity and strictly forbidden in the usual case. Indeed, let us consider the correction to the wave function of the ground state of the hydrogen-like atom $|\Psi_0\rangle \equiv |1, 0, 0\rangle = R_{10}(r)Y_{00} = e^{-r}/\sqrt{\pi}$ due to the perturbation:

$$|\tilde{\Psi}_0\rangle = |\Psi_0\rangle + \sum_{n,l,m} \frac{\langle n, l, m | W | 1, 0, 0 \rangle}{E_n - E_0} |n, l, m\rangle \tag{5}$$

where n, l, m are the usual principal, orbital and magnetic quantum numbers, and the energy $E_n = mQ^2e^2/n^2$.

Using the standard definition of the spherical harmonics Y_{lm} (see e.g. [13]), one can write

$$x = r\sqrt{\frac{2\pi}{3}}(Y_{11} - Y_{1-1}); \quad y = -ir\sqrt{\frac{2\pi}{3}}(Y_{11} + Y_{1-1}); \tag{6}$$

$$x^2 + y^2 = \frac{2}{3}r^2 \left(1 + \sqrt{\frac{4\pi}{5}}Y_{20} \right).$$

So, the action of the perturbation operator (2) on the ground state wave function gives

$$\begin{aligned}
W_1|\Psi_0\rangle &\approx \frac{\mu}{\sqrt{6}mR^2}R_{10}((ivt+b)Y_{11} + (ivt-b)Y_{1-1}); \\
W_2|\Psi_0\rangle &\approx \frac{\mu^2}{2mR^2}R_{10}Y_{00} - \frac{\mu^2}{\sqrt{6}mR^4}rR_{10}((vt-ib)Y_{11} - (vt+ib)Y_{1-1}) + \\
&+ \frac{\mu^2}{3mR^4}r^2R_{10} \left(Y_{00} + \frac{1}{\sqrt{5}}Y_{20} \right). \tag{7}
\end{aligned}$$

Thus, we have nonzero matrix elements of the perturbation operator

$$\begin{aligned}
\langle n, 0, 0 | W | \Psi_0 \rangle &\approx \frac{\mu^2}{2mR^2} I_n[2] + \frac{\mu^2}{3mR^4} I_n[4] + O(\varepsilon^3); \\
\langle n, 2, 0 | W | \Psi_0 \rangle &\approx \frac{\mu^2}{3\sqrt{5}mR^4} I_n[4] + O(\varepsilon^3); \\
\langle n, 1, 1 | W | \Psi_0 \rangle &\approx \frac{\mu}{\sqrt{6}mR^2} \left[I_n[2](b + ivt) + \frac{\mu}{R^2} I_n[3](vt - ib) \right] + O(\varepsilon^3); \\
\langle n, 1, -1 | W | \Psi_0 \rangle &\approx \frac{\mu}{\sqrt{6}mR^2} \left[I_n[2](ivt - b) + \frac{\mu}{R^2} I_n[3](ib + vt) \right] + O(\varepsilon^3), \quad (8)
\end{aligned}$$

where the radial integrals are

$$I_n[k] \equiv \int_0^\infty dr r^k R_{n0} R_{10}$$

In particular, using these expressions, one can calculate the first order correction to the energy of the hydrogen atom ground state

$$\Delta E_0 = \langle \Psi_0 | W | \Psi_0 \rangle \approx \frac{\mu^2}{2mR^2} + O(\varepsilon^3). \quad (9)$$

This result seems to be quite natural. Inded, one can write (9) as $\Delta E_0 \approx kH$, where $k = e^2 g / 2m \sim e / 2m$ is the classical magnetic moment of the hydrogen atom and $H = g / R^2$ is the Coulomb magnetic field of a monopole (here we used the charge quantization condition).

Taking into account the Eqs.(8), we can write the expression for the perturbed ground state wave function

$$\begin{aligned}
|\tilde{\Psi}_0\rangle &= |\Psi_0\rangle + \sum_{n=1}^{\infty} \frac{\langle n, 0, 0 | W | 1, 0, 0 \rangle}{E_n - E_0} |n, 0, 0\rangle + \sum_{n=2}^{\infty} \frac{\langle n, 1, 1 | W | 1, 0, 0 \rangle}{E_n - E_0} |n, 1, 1\rangle \\
&+ \sum_{n=2}^{\infty} \frac{\langle n, 1, -1 | W | 1, 0, 0 \rangle}{E_n - E_0} |n, 1, -1\rangle + \sum_{n=2}^{\infty} \frac{\langle n, 2, 0 | W | 1, 0, 0 \rangle}{E_n - E_0} |n, 2, 0\rangle \\
&= |\Psi_0\rangle + \frac{\mu^2}{2mR^2} \sum_{n=1}^{\infty} \frac{I_n[2]}{E_n - E_0} |n, 0, 0\rangle + \frac{\mu^2}{3mR^4} \sum_{n=1}^{\infty} \frac{I_n[4]}{E_n - E_0} |n, 0, 0\rangle \\
&+ \frac{\mu}{\sqrt{6}mR^2} \sum_{n=2}^{\infty} \frac{1}{E_n - E_0} \left(I_n[2](b - ivt) + \frac{\mu}{R^2} I_n[3](vt - ib) \right) |n, 1, 1\rangle \\
&+ \frac{\mu}{\sqrt{6}mR^2} \sum_{n=2}^{\infty} \frac{1}{E_n - E_0} \left(I_n[2](ivt - b) + \frac{\mu}{R^2} I_n[3](vt + ib) \right) |n, 1, -1\rangle \\
&+ \frac{\mu^2}{3\sqrt{5}mR^2} \sum_{n=2}^{\infty} \frac{I_n[4]}{E_n - E_0} |n, 2, 0\rangle + O(\varepsilon^3). \quad (10)
\end{aligned}$$

It is clear from Eq.(10) that in the presence of a monopole the hydrogen-like atom has nonzero electric dipole moment

$$d = e < \tilde{\Psi}_0 | \mathbf{r} | \tilde{\Psi}_0 > = e < \Psi_0 | \mathbf{r} | \Delta \Psi_0 > + e < \Delta \Psi_0 | \mathbf{r} | \Psi_0 > \quad (11)$$

Indeed, taking into account (10), one can obtain

$$\begin{aligned} < \Psi_0 | z | \Delta \Psi_0 > &= 0; \\ < \Psi_0 | x + iy | \Delta \Psi_0 > &\approx \frac{\mu}{3mR^2} (ivt - b) \sum_{n=2}^{\infty} \frac{I_n[3] I_n[2]}{E_n - E_0} \\ &\quad + \frac{\mu^2}{3mR^4} (vt + ib) \sum_{n=2}^{\infty} \frac{(I_n[3])^2}{E_n - E_0} + O(\varepsilon^3); \\ < \Psi_0 | x - iy | \Delta \Psi_0 > &\approx \frac{\mu}{3mR^2} (ivt + b) \sum_{n=2}^{\infty} \frac{I_n[3] I_n[2]}{E_n - E_0} \\ &\quad + \frac{\mu^2}{3mR^4} (vt - ib) \sum_{n=2}^{\infty} \frac{(I_n[3])^2}{E_n - E_0} + O(\varepsilon^3). \end{aligned} \quad (12)$$

Then we have

$$< \tilde{\Psi}_0 | z | \tilde{\Psi}_0 > = 0; \quad (13)$$

$$\begin{aligned} < \tilde{\Psi}_0 | x + iy | \tilde{\Psi}_0 > &= - < \tilde{\Psi}_0 | x - iy | \tilde{\Psi}_0 > \\ &= - \frac{2\mu b}{3mR^2} \sum_{n=2}^{\infty} \frac{I_n[3] I_n[2]}{E_n - E_0} + \frac{2\mu^2 vt}{3mR^4} \sum_{n=2}^{\infty} \frac{(I_n[3])^2}{E_n - E_0} + O(\varepsilon^3). \end{aligned} \quad (14)$$

So, the magnetic monopole external field leads to the appearance of a nonzero electric dipole moment of the hydrogen atom which, as expected, is proportional to the product of the charges of the monopole and the electron.

To calculate the radial integrals $I_n[k]$ we use the expression for the radial functions

$$R_{n1} = \frac{2}{3} \frac{\sqrt{n(n^2 - 1)}}{n^3} r e^{-r/n} {}_1F_1(-n + 2, 4; \frac{2r}{n}); \quad R_{10} = 2e^{-r} \quad (15)$$

where ${}_1F_1(-N, a, x)$ is the standard confluent hypergeometric function. Taking account its well-known properties [14], it is easy to calculate those integrals in explicit form:

$$I_n[2] = \int_0^{\infty} dr r^2 R_{n1} R_{10} = 2^3 \frac{n \sqrt{n(n^2 - 1)}}{(n + 1)^4} {}_2F_1(-n + 2, 4, 4, \frac{2}{n + 1}); \quad (16)$$

$$I_n[3] = \int_0^{\infty} dr r^3 R_{n1} R_{10} = 2^5 \frac{n^2 \sqrt{n(n^2 - 1)}}{(n + 1)^5} {}_2F_1(-n + 2, 5, 4, \frac{2}{n + 1}); \quad (17)$$

Thus

$$\begin{aligned}\sum_{n=2}^{\infty} \frac{I_n[3]I_n[2]}{E_n - E_0} &= \frac{2^9}{me^2Q^2} \sum_{n=2}^{\infty} \frac{n^6}{(n+1)^9} {}_2F_1(-n+2, 5, 4, \frac{2}{n+1}) {}_2F_1(-n+2, 4, 4, \frac{2}{n+1}); \\ \sum_{n=2}^{\infty} \frac{(I_n[3])^2}{E_n - E_0} &= \frac{2^{11}}{me^2Q^2} \sum_{n=2}^{\infty} \frac{n^7}{(n+1)^{10}} \left({}_2F_1(-n+2, 5, 4, \frac{2}{n+1}) \right)^2.\end{aligned}\quad (18)$$

After some numerical calculations one can obtain in the first order of perturbation theory

$$\left| \langle \tilde{\Psi}_0 | x - iy | \tilde{\Psi}_0 \rangle \right| = \left| \langle \tilde{\Psi}_0 | x + iy | \tilde{\Psi}_0 \rangle \right| \approx \frac{b}{(meQ)^2} \frac{\mu}{R^2} + O(\varepsilon^2) = 2b \frac{\Delta E_0}{E_0} + O(\varepsilon^2). \quad (19)$$

Note, that the appearance of the electric dipole moment (19) of the hydrogen atom in the monopole presence is not connected with the well known extra angular momentum in the charge-monopole system.

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References

- [1] Amaldi E. et.al., "Search for Dirac magnetic poles" Report 63-13, CERN (1963)
- [2] Trefil J.S., Nucl.Phys. **B203** (1982) 501.
- [3] Tiktopoulos G., Phys.Lett. **125B** (1983) 156.
- [4] Tolkachev E.A., Tomil'chik L.M., and Shnir Ya.M., Sov.J.Nucl.Phys. **52** (1990) 916.
- [5] Fleischer R. and Walker R., Phys.Rev.Lett. **35** (1975) 1412.
- [6] Drell S. et.al., Phys.Rev.Lett. **50** (1983) 644.
- [7] Sachs M., Ann. of Phys., **6** (1959) 209.
- [8] Tomil'chik L.M., Sov.J. of Exp. and Theor. Phys., **44** (1963) 160.
- [9] Tolkachev E.A., Tomil'chik L.M. and Shnir Ya.M., Sov.J.Nucl.Phys. **38** (1983) 320.

- [10] Tolkachev E.A., Tomil'chik L.M. and Shnir Ya.M., Int. J. Mod. Phys. **A7** (1992) 3747.
- [11] Barut A.O., Knyazev M.A., Tolkachev E.A. and Shnir Ya.M., Physica Scripta **49** (1994) 513.
- [12] Kovalevich S.G., Tolkachev E.A. and Shnir Ya.M., Physica Scripta **53** (1996) 51.
- [13] Landau L.D., Lifshitz E.M., Quantum Mechanics, *3rd* ed., Pergamon, Oxford, 1977.
- [14] Gradshteyn I.S. and Ryzhik I.M. Table of Integrals, Series and Products, London, Academy Press, 1980.